



PARP Research Group

Problem:

Bundle Adjustment (BA) is usually the main performance bottleneck in incremental SfM.

Proposal:

Complement BA with a previous gobal epipolar adjustment (GEA): GEA optimization provides a map very close to the BA optimal configuration, requiring less BA iterations to reach the optimal.

Simplified algebraic epipolar cost has computational and qualitative advantages for fast/real-time performance. Prior data normalization approximates the algebraic cost to geometric; in practice algebraic cost offers a competitive trade-off between *performance time* and *error* reduction.

1. Global epipolar cost error

$$C_{ege} = \sum_{p} \sum_{i[p]} \sum_{j[p]} d(x_i^{(p)}, E_{ij} \; x_j^{(p)})$$

Where:

$$E_{ij} = R_i \left[\frac{T_j - T_i}{||T_j - T_i||} \right]_{\times} R_j^T$$

 $\mathbf{E}_{ii} \rightarrow \mathbf{E}_{ii}$ sential matrix for epipolar geometry between views *i* and *j*.

 $p \rightarrow 2D$ feature tracking for 3D point.

 $\mathbf{R}_i, \mathbf{T}_i \rightarrow \mathbf{R}$ otation matrix and center vector for the *i*-th camera.

2. Global epipolar adjustment

SfM map optimization:

- (1) Initialization: computation of new view-pair epipolar constraints.
- (2) Levenberg-Marquart optimization: find optimal value for the epipolar cost C_{ege} (iterative, on cameras only).
- (3) Structure computation: linear estimation when 3D points needed.
- (4) Bundle Adjustment: final polishing of geometric reprojection error.

Reduced Epipolar Cost for Accelerated Incremental SfM¹ A.L. Rodríguez², P.E.López-de-Teruel², A.Ruiz³ {*alrl1*, *pedroe*, *aruiz*}@*um.es*

3. Reduced measurement matrix

Algebraic cost (computational advantages):

$$d(x_i, E_{ij}x_j) = x_i^T E_{ij}x_j = m_{ij}^T e_{ij}$$

where $e_{ii} \rightarrow vectorization of matrix E_{ii}$. Error C_{eve} becomes:

$$C_e = \sum_{i \neq j} \|M_{ij} e_{ij}\|^2 = \sum_{i \neq j} e_{ij}^T M_{ij}^T M_{ij} e_{ij}$$

Cholesky decomposition of matrix $\Omega = \mathbf{M}^{T}\mathbf{M}$ provides upper triangular matrix L_{9x9} equivalent to M_{nx9} for evaluation of cost function C_{c} :

$$M^T M = \Omega = L^T L$$

Each L_{ii} is computed once for each view pair, and reused in subsequent iterative SfM steps substituting matrix M_{ny9} in the evaluation of C_{c} .

4. Levenberg-Marguardt optimization

- \rightarrow 3D point coordinates not needed: only 2D view-pair relations. No Schur complement required to eliminate 3D point parameters from the LM second level system.
- \rightarrow Degrees of freedom of the cost function: 6 × #cams.
- \rightarrow GEA Hessian sparsity structure is equivalent to that obtained in BA.













5. Results

Performance in milliseconds. Solve step uses CHOLMOD (sSBA) and MKL (GEA):

	Set size		sSBA		GEA			
Dataset	#Cams	#Pts	Rest	Solve	RM	Rest	Solve	LT
trafalgar	256	65k	797	997	282	105	331	957
lubrovnik	356	227k	7024	4351	2977	479	1567	3442
venice	245	199k	6596	604	2837	285	738	3060
venice	427	310k	12387	14480	4708	736	2445	4871

$\mathbf{RM} \rightarrow \mathbf{Reduced}$ meas
matrix estimation.
Rest + Solve \rightarrow Iteratio

 $LT \rightarrow Linear$ estimation of 3D structure (points).

Optimization of unstructured data-sets:

Drift error correction using loop closing information:



Also, convergence basin of GEA is significantly larger, able to recover the correct solution from surprisingly large initial reprojection errors (see Fig. 2 in paper).

URL: http://perception.inf.um.es/gea

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2. Departamento de Ingeniería y Tecnología de Computadores, Universidad de Murcia. 3. Departamento de Informática y Sistemas, Universidad de Murcia.

